



Let's study.

To construct a triangle, if following information is given.

- Base, an angle adjacent to the base and sum of lengths of two remaining sides.
- Base, an angle adjacent to the base and difference of lengths of remaining two sides.
- Perimeter and angles adjacent to the base.



Let's recall.

In previous standard we have learnt the following triangle constructions.

- * To construct a triangle when its three sides are given.
- * To construct a triangle when its base and two adjacent angles are given.
- * To construct a triangle when two sides and the included angle are given.
- * To construct a right angled triangle when its hypotenuse and one side is given.

Perpendicular bisector Theorem

- Every point on the perpendicular bisector of a segment is equidistant from its end points.
- Every point equidistant from the end points of a segment is on the perpendicular bisector of the segment.

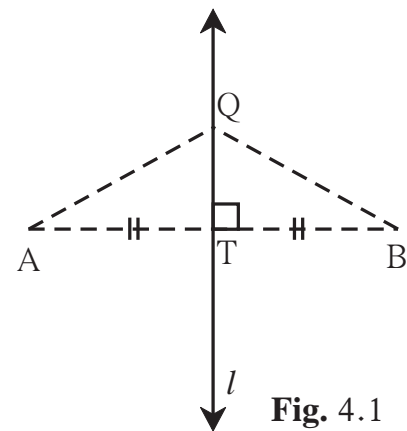


Fig. 4.1



Let's learn.

Constructions of triangles

To construct a triangle, three conditions are required. Out of three sides and three angles of a triangle two parts and some additional information about them is given, then we can construct a triangle using them.

We frequently use the following property in constructions.

If a point is on two different lines then it is the intersection of the two lines.

Construction I

To construct a triangle when its base, an angle adjacent to the base and the sum of the lengths of remaining sides is given.

Ex. Construct $\triangle ABC$ in which $BC = 6.3$ cm, $\angle B = 75^\circ$ and $AB + AC = 9$ cm.

Solution : Let us first draw a rough figure of expected triangle.

Explanation : As shown in the rough figure, first we draw seg $BC = 6.3$ cm of length. On the ray making an angle of 75° with seg BC , mark point D such that

$$BD = AB + AC = 9 \text{ cm}$$

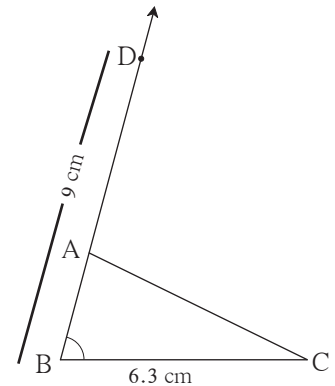
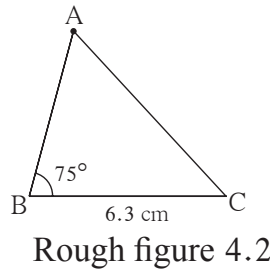
Now we have to locate point A on ray BD .

$$BA + AD = BA + AC = 9$$

$$\therefore AD = AC$$

\therefore point A is on the perpendicular bisector of seg CD .

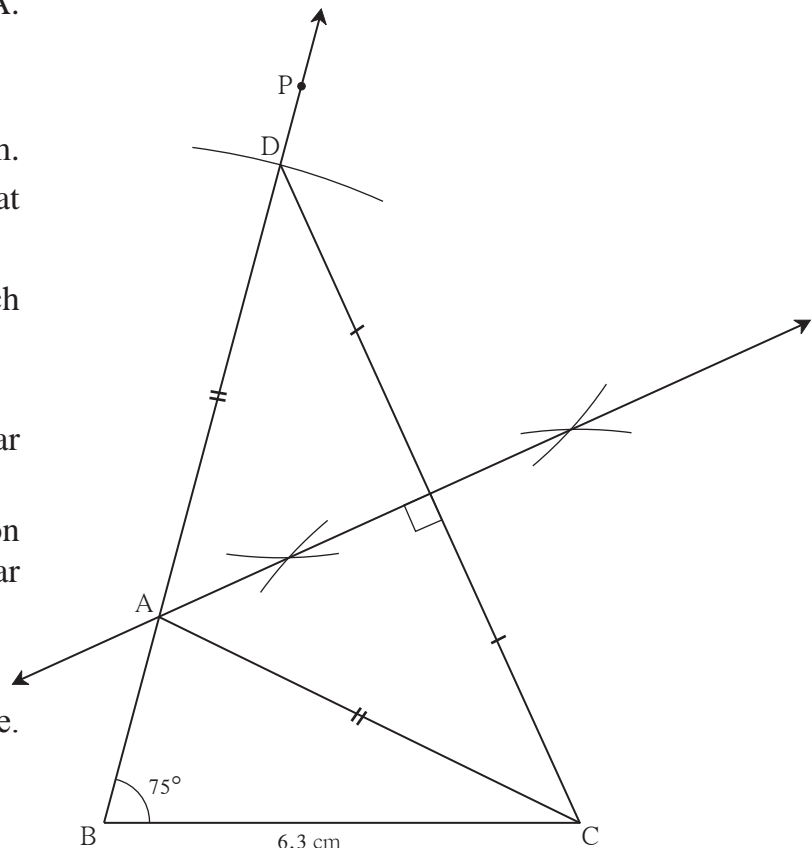
\therefore the point of intersection of ray BD and the perpendicular bisector of seg CD is point A .



Steps of construction

- (1) Draw seg BC of length 6.3 cm.
- (2) Draw ray BP such that $m\angle PBC = 75^\circ$.
- (3) Mark point D on ray BP such that $d(B,D) = 9$ cm
- (4) Draw seg DC .
- (5) Construct the perpendicular bisector of seg DC .
- (6) Name the point of intersection of ray BP and the perpendicular bisector of CD as A .
- (7) Draw seg AC .

$\triangle ABC$ is the required triangle.



Fair fig. 4.4

Practice set 4.1

1. Construct $\triangle PQR$, in which $QR = 4.2$ cm, $m\angle Q = 40^\circ$ and $PQ + PR = 8.5$ cm
2. Construct $\triangle XYZ$, in which $YZ = 6$ cm, $XY + XZ = 9$ cm. $\angle XYZ = 50^\circ$
3. Construct $\triangle ABC$, in which $BC = 6.2$ cm, $\angle ACB = 50^\circ$, $AB + AC = 9.8$ cm
4. Construct $\triangle ABC$, in which $BC = 3.2$ cm, $\angle ACB = 45^\circ$ and perimeter of $\triangle ABC$ is 10 cm

Construction II

To construct a triangle when its base, angle adjacent to the base and difference between the remaining sides is given.

Ex (1) Construct $\triangle ABC$, such that $BC = 7.5$ cm, $\angle ABC = 40^\circ$, $AB - AC = 3$ cm.

Solution : Let us draw a rough figure.

Explanation : $AB - AC = 3$ cm $\therefore AB > AC$

Draw seg BC . We can draw the ray BL such that $\angle LBC = 40^\circ$. We have to locate point A on ray BL . Take point D on ray BL such that $BD = 3$ cm.

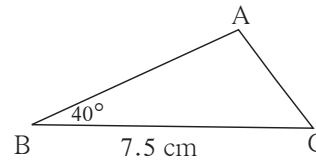
Now, $B-D-A$ and $BD = AB - AD = 3$.

It is given that $AB - AC = 3$

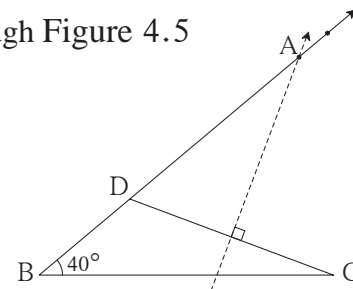
$\therefore AD = AC$

\therefore point A is on the perpendicular bisector of seg DC .

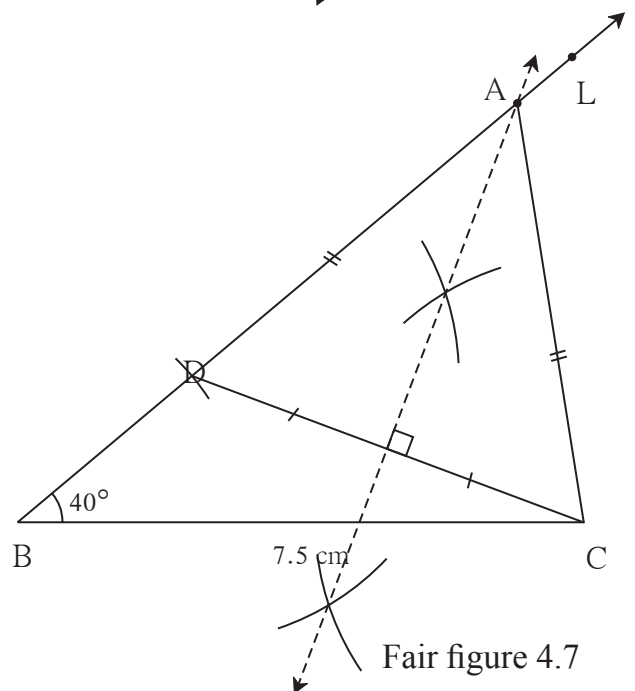
\therefore point A is the intersection of ray BL and the perpendicular bisector of seg DC .



Rough Figure 4.5



Rough figure 4.6



Fair figure 4.7

Steps of construction

- (1) Draw seg BC of length 7.5 cm.
- (2) Draw ray BL such that $\angle LBC = 40^\circ$
- (3) Take point D on ray BL such that $BD = 3$ cm.
- (4) Construct the perpendicular bisector of seg CD .
- (5) Name the point of intersection of ray BL and the perpendicular bisector of seg CD as A .
- (6) Draw seg AC .
 $\triangle ABC$ is required triangle.

Ex. 2 Construct $\triangle ABC$, in which side $BC = 7$ cm, $\angle B = 40^\circ$ and $AC - AB = 3$ cm.

Solution : Let us draw a rough figure.

seg $BC = 7$ cm. $AC > AB$.

We can draw ray BT such that

$\angle TBC = 40^\circ$

Point A is on ray BT . Take point D on opposite ray of ray BT such that

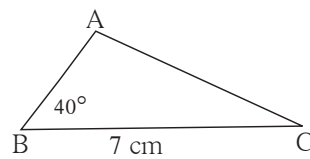
$BD = 3$ cm.

Now $AD = AB + BD = AB + 3 = AC$

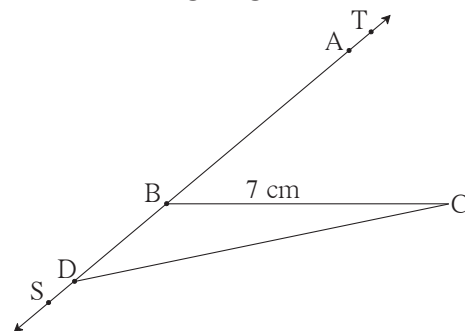
($\because AC - AB = 3$ cm.)

$\therefore AD = AC$

\therefore point A is on the perpendicular bisector of seg CD .



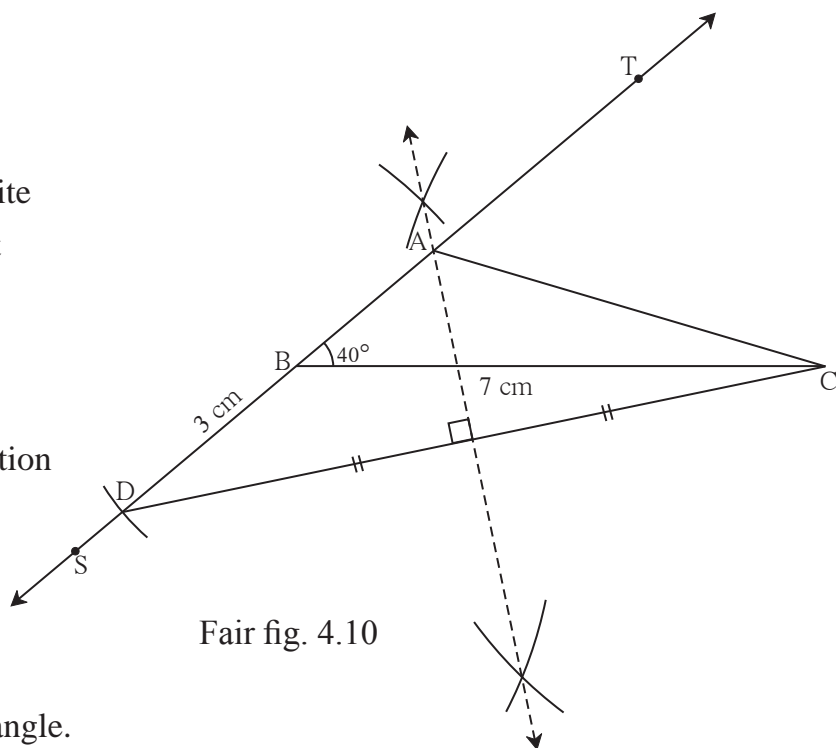
Rough figure 4.8



Rough figure 4.9

Steps of construction

- (1) Draw BC of length 7 cm.
- (2) Draw ray BT such that $\angle TBC = 40^\circ$
- (3) Take point D on the opposite ray BS of ray BT such that $BD = 3$ cm.
- (4) Construct perpendicular bisector of seg DC .
- (5) Name the point of intersection of ray BT and the perpendicular bisector of DC as A .
- (6) Draw seg AC .
 $\triangle ABC$ is the required triangle.



Fair fig. 4.10

Practice set 4.2

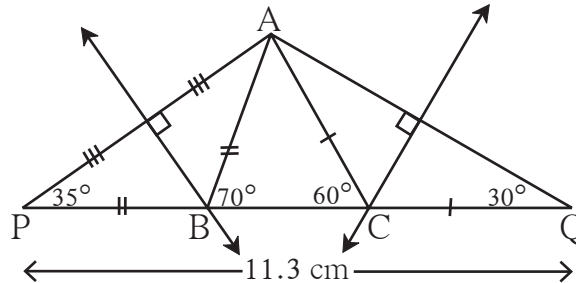
1. Construct $\triangle XYZ$, such that $YZ = 7.4$ cm, $\angle XYZ = 45^\circ$ and $XY - XZ = 2.7$ cm.
2. Construct $\triangle PQR$, such that $QR = 6.5$ cm, $\angle PQR = 40^\circ$ and $PQ - PR = 2.5$ cm.
3. Construct $\triangle ABC$, such that $BC = 6$ cm, $\angle ABC = 100^\circ$ and $AC - AB = 2.5$ cm.

Construction III

To construct a triangle, if its perimeter, base and the angles which include the base are given.

Ex. Construct $\triangle ABC$ such that $AB + BC + CA = 11.3$ cm, $\angle B = 70^\circ$, $\angle C = 60^\circ$.

Solution : Let us draw a rough figure.



Rough Fig. 4.11

Explanation : As shown in the figure, points P and Q are taken on line BC such that,

$$PB = AB, \quad CQ = AC$$

$$\therefore PQ = PB + BC + CQ = AB + BC + AC = 11.3 \text{ cm.}$$

Now in $\triangle PBA$, $PB = BA$

$$\therefore \angle APB = \angle PAB \text{ and } \angle APB + \angle PAB = \text{exterior angle } ABC = 70^\circ$$

.....theorem of remote interior angles

$$\therefore \angle APB = \angle PAB = 35^\circ \quad \text{Similarly, } \angle CQA = \angle CAQ = 30^\circ$$

Now we can draw $\triangle PAQ$, as its two angles and the included side is known.

Since $BA = BP$, point B lies on the perpendicular bisector of seg AP.

Similarly, $CA = CQ$, therefore point C lies on the perpendicular bisector of seg AQ

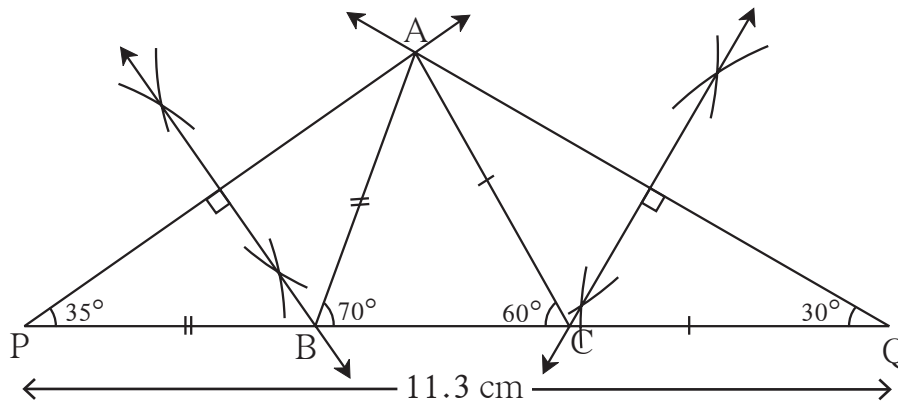
\therefore by constructing the perpendicular bisectors of seg AP and AQ we can get points

B and C, where the perpendicular bisectors intersect line PQ.

Steps of construction

- (1) Draw seg PQ of 11.3 cm length.
- (2) Draw a ray making angle of 35° at point P.
- (3) Draw another ray making an angle of 30° at point Q.
- (4) Name the point of intersection of the two rays as A.
- (5) Draw the perpendicular bisector of seg AP and seg AQ. Name the points as B and C respectively where the perpendicular bisectors intersect line PQ.
- (6) Draw seg AB and seg AC.

$\triangle ABC$ is the required triangle.



Final Fig. 4.12

Practice set 4.3

1. Construct ΔPQR , in which $\angle Q = 70^\circ$, $\angle R = 80^\circ$ and $PQ + QR + PR = 9.5$ cm.
2. Construct ΔXYZ , in which $\angle Y = 58^\circ$, $\angle X = 46^\circ$ and perimeter of triangle is 10.5 cm.
3. Construct ΔLMN , in which $\angle M = 60^\circ$, $\angle N = 80^\circ$ and $LM + MN + NL = 11$ cm.

Problem set 4

1. Construct ΔXYZ , such that $XY + XZ = 10.3$ cm, $YZ = 4.9$ cm, $\angle XYZ = 45^\circ$.
2. Construct ΔABC , in which $\angle B = 70^\circ$, $\angle C = 60^\circ$, $AB + BC + AC = 11.2$ cm.
3. The perimeter of a triangle is 14.4 cm and the ratio of lengths of its side is 2 : 3 : 4. Construct the triangle.
4. Construct ΔPQR , in which $PQ - PR = 2.4$ cm, $QR = 6.4$ cm and $\angle PQR = 55^\circ$.



ICT Tools or Links

Do constructions of above types on the software Geogebra and enjoy the constructions. The third type of construction given above is shown on Geogebra by a different method. Study that method also.

