9. DIFFERENTIATION

Let's study.

- The meaning of rate of change.
- Meaning of derivative and the formula associated with it.
- Derivatives of some standard functions.
- Some applications of derivatives

Let's recall.

- Real valued functions on R.
- Limits of functions.
- Continuity of a function at a point and over an interval.

The meaning of rate of change.

Suppose we are traveling in a car from Mumbai to Pune. We are displacing ourselves from the origin (Mumbai) from time to time. We know that the speed of the car

$$= \frac{\text{Distance travelled by the car}}{\text{Time taken to travel the distance}}$$

But at different times the speed of the car can be different. It is the ratio of, a very small distance travelled and very small time taken to travel that distance. The limit of this ratio, as the time interval tends to zero is the speed of the car at that time. This process of obtaining the speed is given by the differentiation of the distance function with respect to time. This is an example of derivative or differentiation which is useful. This measures how quickly the position of the object changes with time.

When we speak of velocity, it is the speed with the direction of movement. In problems with no change in direction, words speed and velocity may be interchanged. The rate of change in a function at a point with respect to the variable is called the derivative of the function at that point. The process of finding a derivative is called differentiation

9.1 Definition of Derivative and differentiability

Let f be a function defined on an open interval containing 'a'. If the limit $\lim\limits_{\delta x \to 0} \frac{f(a+\delta x)-f(a)}{\delta x}$ exists, then f is said to be differentiable at $x = a$. This limit is denoted by $f'(a)$ and is given by $f'(a) = \lim\limits_{\delta x \to 0} \frac{f(a+\delta x)-f(a)}{\delta x}$ $f'(a)$ is also called the derivative of 'f' at a.

If $y = f(x)$ is the functional and if the limiting value of the function exists, then that limiting value is called the derivative of the function and it is symbolically represented as,

$$\frac{dy}{dx} = f'(x) = y'$$

Let's Note.

(1) If y is a differentiable function of x then

$$\lim_{\delta x \to 0}\left(\frac{\delta y}{\delta x}\right) = \frac{dy}{dx} \text{ and}$$

$$\lim_{\delta x \to 0}\left(\frac{f(x+\delta x)-f(x)}{\delta x}\right) = f'(x) = y' = y_1$$

9.2 Derivative by the method of first principle.

The process of finding the derivative of a function using the definition of derivative is known as derivative from the first principle. Just for the sake of convenience δx is replaced by h.

If $y = f(x)$ is the given function, then the derivative of y *w.r.t.x* is represented as

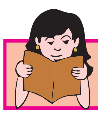
$$\frac{dy}{dx} = f'(x) = \lim_{h \to o}\left(\frac{f(x+h) - f(x)}{h}\right)$$

9.3 Notations for derivatives :

If $y = f(x)$ is a given function then the derivative of $f(x)$ is represented in many different ways. Many mathematicians have used different notations for derivatives.

Most commonly used is the Leibniz's notation. The symbols dx, dy and $\frac{dy}{dx}$ were introduced by **Gottfried Wilhelm Leibnitz**.

Note : If we consider the graph of a function in XY-plane, then the derivative of the function f at a point 'a' , is the slope of the tangent line to the curve $y = f(x)$ at $x = a$. (We are going to study this in detail in next level)



Let's learn.

Derivatives of some standard functions

Ex. (1) Find the derivative of x^n w. r. t. x. ($n \in N$)

Solution :

Let $f(x) = x^n$

$f(x + h) = (x + h)^n$

By the method of first principle,

$$f'(x) = \lim_{h \to o}\left(\frac{f(x+h) - f(x)}{h}\right)$$

$$= \lim_{h \to 0}\left(\frac{(x+h)^n - x^n}{x+h-x}\right)$$

Let $x + h = y$, as $h \to 0, x \to y$

$$f'(x) = \lim_{x \to y}\left(\frac{y^n - x^n}{y - x}\right)$$

$$f'(x) = nx^{n-1} \quad \ldots\ldots\ldots \left[\because \lim_{x \to a}\left(\frac{x^n - a^n}{x - a}\right) = na^{n-1}\right]$$

Ex. (2) Find the derivative of a^x w. r. t. $x.(a > 0)$

Solution :

Let $f(x) = a^x$

$f(x + h) = a^{x+h}$

By the method of first principle,

$$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$$

$$= \lim_{h \to 0}\left(\frac{a^{(x+h)} - a^x}{h}\right)$$

$$= \lim_{h \to 0}\left(\frac{a^x(a^h - 1)}{h}\right)$$

$$= a^x \lim_{h \to 0}\left(\frac{a^h - 1}{h}\right)$$

$$f'(x) = a^x \log a \quad \ldots\ldots\ldots \left[\because \lim_{x \to 0}\left(\frac{a^x - 1}{x}\right) = \log a\right]$$

Try the following

(1) If $f(x) = \frac{1}{x^n}$, then prove that $f'(x) = \frac{-n}{x^{n+1}}$

(2) If $f(x) = e^x$, then prove that $f'(x) = e^x$

SOLVED EXAMPLES

EX. 1. Find the derivatives of the following by using the method of first principle

(i) $\sqrt{x}$ (ii) 4^x (iii) $\log x$

Solution :

(i) Let $f(x) = \sqrt{x}$

$f(x+h) = \sqrt{x+h}$

By the method of first principle,

$$f'(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{\sqrt{(x+h)}-\sqrt{x}}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{\sqrt{(x+h)}-\sqrt{x}}{h}\times\frac{\sqrt{(x+h)}+\sqrt{x}}{\sqrt{(x+h)}+\sqrt{x}}\right)$$

$$= \lim_{h\to 0}\left(\frac{x+h-x}{h\left(\sqrt{(x+h)}+\sqrt{x}\right)}\right)$$

$$= \lim_{h\to 0}\left(\frac{h}{h\left(\sqrt{(x+h)}+\sqrt{x}\right)}\right)$$

$$= \lim_{h\to 0}\left(\frac{1}{\sqrt{x+h}+\sqrt{x}}\right) \text{ .... (As } h\to 0, h\neq 0)$$

$$= \frac{1}{\sqrt{x+0}+\sqrt{x}}$$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

(ii) Let $f(x) = 4^x$

$f(x+h) = 4^{x+h}$

By method of first principle,

$$f'(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

$$f'(x) = \lim_{h\to 0}\left(\frac{4^{x+h}-4^x}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{4^x(4^h-1)}{h}\right)$$

$$= 4^x \lim_{h\to 0}\left(\frac{4^h-1}{h}\right)$$

$$f'(x) = 4^x\log 4 \text{ ....} \left[\because \lim_{x\to 0}\left(\frac{a^x-1}{x}\right)=\log a\right]$$

(iii) Let $f(x) = \log x$

$f(x+h) = \log(x+h)$

$$f'(x) = \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\log(x+h)-\log(x)}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\log\left(\frac{x+h}{x}\right)}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\log\left(1+\frac{h}{x}\right)}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\log\left(1+\frac{h}{x}\right)}{\frac{h}{x}}\right]\times\frac{1}{x}$$

$$= (1)\left(\frac{1}{x}\right) \text{.... } \because \lim_{x\to 0}\left[\frac{\log(1+x)}{x}\right]=1$$

$$= \frac{1}{x}$$

$$\therefore f'(x) = \frac{1}{x}$$

EX. 2. Find the derivative of $x^2 + x + 2$ at $x = -3$

Let $f(x) = x^2 + x + 2$

For $x = -3$, $f(-3) = (-3)^2 - 3 + 2 = 9 - 3 + 2 = 8$

$f(-3+h) = (-3+h)^2 + (-3+h) + 2$
$= h^2 - 6h + 9 - 3 + h + 2 = h^2 - 5h + 8$

By method of first principle,

$$f'(a) = \lim_{h\to 0}\left(\frac{f(a+h)-f(a)}{h}\right)$$

$$\therefore f'(-3) = \lim_{h\to 0}\left(\frac{f(-3+h)-f(-3)}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{h^2-5h+8-8}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{h(h-5)}{h}\right)$$

$$= \lim_{x\to 0}(h-5) = -5 \text{ (As } h\to 0,\ h\neq 0)$$

$\therefore f'(-3) = -5$

9.4 Rules of Differentiation (without proof)

Theorem 1. Derivative of Sum of functions.

If u and v are differentiable functions of x such that $y = u + v$ then $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

Theorem 2. Derivative of Difference of functions.

If u and v are differentiable functions of x such that $y = u - v$ then $\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$

Corollary 1 : If u, v, w.... are differentiable functions of x such that $y = k_1 u \pm k_2 v \pm k_3 w \pm$......... where k_1, k_2, k_3 .... are constants then

$$\frac{dy}{dx} = k_1\frac{du}{dx} \pm k_2\frac{dv}{dx} \pm k_3\frac{dw}{dx} \ ....$$

Theorem 3. Derivative of Product of functions.

If u and v are differentiable functions of x such that $y = u.v$ then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

Corollary 2 : If u, v and w are differentiable functions of x such that $y = u.v.w$ then

$$\frac{dy}{dx} = uv\frac{dw}{dx} + uw\frac{dv}{dx} + vw\frac{du}{dx}$$

Theorem 4. Derivative of Quotient of functions.

If u and v are differentiable functions of x such that $y = \frac{u}{v}$ where $v \neq 0$ then $\frac{dy}{dx} = \frac{v.\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$

Derivatives of Algebraic Functions

Sr. No.	y	dy/dx
	$f(x)$	$f'(x)$
01	x^n	nx^{n-1}
02	$\frac{1}{x^n}$	$-\frac{n}{x^{n+1}}$
03	$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
04	c	0

SOLVED EXAMPLES

Ex. 1: Differentiate the following functions *w.r.t.x*.

i) $y = x^4 - 2x^3 + \sqrt{x} - \frac{3}{x^2} - 8$

ii) $y = (2x + 3)(x^3 - 7x + 4)$

iii) $x \log x$

iv) $y = \log x\,(e^x + x)$

Solution :

1) $y = x^4 - 2x^3 + \sqrt{x} - \dfrac{3}{x^2} - 8$

$\therefore\ y = x^4 - 2x^3 + x^{1/2} - 3x^{-2} - 8$

Differentiate *w.r.t.x*.

$$\frac{dy}{dx} = \frac{d}{dx}(x^4) - 2\frac{d}{dx}(x^3) + \frac{d}{dx}(x^{1/2})$$

$$-3\frac{d}{dx}(x^{-2}) - \frac{d}{dx}(8) \text{ .... (by rule 1)}$$

$$= 4x^3 - 2(3x^2) + (\frac{1}{2}x^{-1/2}) - 3(-2x^{-3}) - 0$$

$$= 4x^3 - 6x^2 + (\frac{1}{2}x^{-1/2}) + 6x^{-3}$$

$$= 4x^3 - 6x^2 + \frac{1}{2\sqrt{x}} + \frac{6}{x^3}$$

ii) $y = (2x + 3)(x^3 - 7x + 4)$

Differentiating *w.r.t.x*

$$\frac{dy}{dx} = (2x + 3)\frac{d}{dx}(x^3 - 7x + 4)$$

$$+ (x^3 - 7x + 4)\ \frac{d}{dx}\ (2x + 3)$$

$$= (2x + 3)(3x^2 - 7) + (x^3 - 7x + 4)\ (2)$$

$$= 6x^3 + 9x^2 - 14x - 21 + 2x^3 - 14x + 8$$

$$= 8x^3 + 9x^2 - 28x - 13$$

iii) $y = x.\log x$

Differentiating *w.r.t.x*

$$\frac{dy}{dx} = x\frac{d}{dx}\ (\log x) + \log x\ \frac{d}{dx}(x)$$

$$= x\frac{1}{x} + \log x.1 \quad = 1 + \log x$$

iv) $y = \log x\ (e^x + x)$

Differentiating *w.r.t.x*

$$\frac{dy}{dx} = \log x\frac{d}{dx}(e^x + x) + (e^x + x)\ \frac{d}{dx}(\log x)$$

$$= \log x\left(\frac{d}{dx}(e^x) + \frac{d}{dx}(x)\right) + (e^x + x)\ \left(\frac{1}{x}\right)$$

$$= \log x(e^x + 1) + (e^x + x)\frac{1}{x}$$

$$= e^x\left(\log x + \frac{1}{x}\right) + \log x + 1$$

2) Differentiate the following functions *w.r.t.x*

i) $y = \dfrac{x^2 + 7x - 9}{x^2 - 1}$

ii) $y = \dfrac{x}{x+3}$

iii) $y = \dfrac{1 + \log x}{x}$

Solution:

i) $y = \dfrac{x^2 + 7x - 9}{x^2 - 1} = \dfrac{N}{D}$, say.

Differentiating *w.r.t.x.*

$$\frac{dy}{dx} = \frac{D.\frac{d}{dx}(N) - N\frac{d}{dx}(D)}{D^2}$$

(by quotient rule)

$$= \frac{(x^2 - 1)(2x + 7) - (x^2 + 7x - 9)(2x)}{(x^2 - 1)^2}$$

$$= \frac{2x^3 - 2x + 7x^2 - 7 - 2x^3 - 14x^2 + 18x}{(x^2 - 1)^2}$$

$$= \frac{-7x^2 + 16x - 7}{(x^2 - 1)^2}$$

ii) $y = \dfrac{x}{x+3}$

Differentiating *w.r.t.x.*

$$\frac{dy}{dx} = \frac{(x+3)\frac{d}{dx}x - x\frac{d}{dx}(x+3)}{(x+3)^2}$$

$$= \frac{(x+3)\ 1 - x(1)}{(x+3)^2}$$

$$= \frac{3}{(x+3)^2}$$

iii) $y = \dfrac{1+\log x}{x}$

Differentiating *w.r.t.x.,* we get

$$\frac{dy}{dx} = \frac{x\frac{d}{dx}(1+\log x) - (1+\log x)\frac{d}{dx}(x)}{x^2}$$

$$= \frac{x\left(0+\frac{1}{x}\right) - (1+\log x)(1)}{x^2}$$

$$= \frac{1-1-\log x}{x^2}$$

$$= \frac{-\log x}{x^2}$$

Derivatives of Logarithmic and Exponential functions

Sr.No.	y	dy/dx
	$f(x)$	$f'(x)$
01	$\log x$	$1/x$
02	e^x	e^x
03	$a^x (a>0)$	$a^x \log a$

EXERCISE 9.1

(I) Find the derivatives of the following functions *w. r. t. x.*

(1) x^{12} (2) x^{-9} (3) $x^{3/2}$

(4) $7x\sqrt{x}$ (5) 3^5

(II) Differentiate the following *w. r. t. x.*

(1) $x^5 + 3x^4$

(2) $x\sqrt{x} + \log x - e^x$

(3) $x^{5/2} + 5x^{7/5}$ (4) $\frac{2}{7}x^{7/2} + \frac{5}{2}x^{2/5}$

(5) $\sqrt{x}\,(x^2 + 1)^2$

(III) Differentiate the following *w. r. t. x*

(1) $x^3 \log x$ (2) $x^{\frac{5}{2}} e^x$

(3) $e^x \log x$ (4) $x^3.3^x$

(IV) Find the derivatives of the following *w. r. t. x.*

(1) $\dfrac{x^2+a^2}{x^2-a^2}$ (2) $\dfrac{3x^2+5}{2x^2-4}$

(3) $\dfrac{\log x}{x^3-5}$ (4) $\dfrac{3e^x-2}{3e^x+2}$

(5) $\dfrac{xe^x}{x+e^x}$

(V) Find the derivatives of the following functions by the first principle.

i) $3x^2 + 4$ ii) $x\sqrt{x}$

iii) $\dfrac{1}{2x+3}$ iv) $\dfrac{x-1}{2x+7}$

SOME APPLICATIONS OF DERIVATIVES:

Supply and **demand** are perhaps one of the most fundamental concepts of economics and it is the backbone of a market economy. **Supply** represents how much the market can offer. The quantity supplied refers to the amount of a certain goods producers are willing to **supply** when receiving a certain price. **Demand** is the quantity of goods or commodity demanded. Similarly **Cost, Revenue and Profit** etc are most commonly used terminologies in business and economics.

DEMAND FUNCTION :

As Demand (D) is a function of Price (P), we can express it as $D = f(P)$.

Marginal Demand (MD):

The rate of change of demand with respect to price is called the **Marginal Demand** (MD).

So, $MD = \dfrac{dD}{dp} = f'(p)$

SUPPLY FUNCTION (S):

Supply (S) is also a function of Price (P), we can express it as $S = g(p)$.

Marginal Supply (MS):

The rate of change of supply with respect to price is called the **Marginal Supply** (MS).

Thus, $MS = \dfrac{dS}{dp} = g'(p)$

TOTAL COST FUNCTION (C):

A Cost (C) function is a mathematical formula used to express the production expenses of number of goods produced. It can be express as $C = f(x)$ where x is the number of goods produced.

Marginal Cost (MC):

The rate of change of Cost with respect to number of goods i.e. x is called **Marginal Cost** (MC).

Therefore, $MC = \dfrac{dC}{dx} = f'(x)$

Average Cost (AC) is the cost of production of each goods. So, $AC = \dfrac{C}{x}$

Revenue and Profit Functions:

If R(x) is the revenue received from the sale of x units of some goods (or commodity), then the derivative, R'(x) is called the Marginal Revenue.

Total Revenue (R):

The Total Revenue is given by R = P.D where P is price and D is quantity of goods demanded.

Average Revenue = $\dfrac{R}{D} = \dfrac{P.D}{D} = P$ i.e. price it self.

Total Profit *(P)* = Revenue – Cost

$P = R - C$

SOLVED EXAMPLES

Ex. (1) The demand D of a goods at price p is given by $D = P^2 + \dfrac{32}{P}$.

Find the marginal demand when price is Rs.2.

Solution : Demand is given by $D = P^2 + \dfrac{32}{P}$

Diff. w. r. t. p.

$$\frac{dD}{dP} = 2P - \frac{32}{P^2}$$

$$\left(\frac{dD}{dP}\right)_{p=4} = 2(4) - \frac{32}{16} = 8 - 2 = 6$$

Ex. (2) The cost C of an output is given as $C = 2x^3 + 20x^2 - 30x + 45$. What is the rate of change of cost when the the output is 2 ?

Solution :

$C = 2x^3 + 20x^2 - 30x + 45$

Diff. w. r. t. x.

$$\frac{dC}{dx} = 6x^2 + 40x - 30$$

$$\left(\frac{dC}{dx}\right)_{x=2} = 6(2)^2 + 40(2) - 30 = 74$$

Average Cost = $AC = \dfrac{C}{x}$

$$= \frac{2x^3 + 20x^2 - 30x + 45}{x}$$

$$= 2x^2 + 20x - 30 + \frac{45}{x}$$

Ex. (3) The demand D for a chocolate is inversely proportional to square of it's price P. It is observed that the demand is 4 when price is 4 per chocolate. Find the marginal demand when the price is 4.

Solution : Given that Demand is inversely proportional to square of the price

i.e $D \propto \frac{1}{P^2}$

$$D = \frac{k}{P^2} \text{ where } k > 0 \quad \ldots\ldots (1)$$

Also , D = 4, when P = 4 , from (1)

$$4 = \frac{k}{4^2} \implies k = 64$$

Equation (1) becomes

$$D = \frac{64}{P^2}$$

Diff. w. r. t. P.

$$\frac{dD}{dP} = -\frac{128}{P^3} \quad \therefore \text{ when } P = 4,$$

$$\left(\frac{dD}{dP}\right)_{P=4} = -\frac{128}{64} = -2$$

Ex (4) The relation between supply S and price P of a commodity is given as $S = 2P^2 + P - 1$.

Find the marginal supply at price 4.

Solution :

Given , $S = 2P^2 + P - 1$

Diff. w. r. t. P.

$$\therefore \frac{dS}{dP} = 4P + 1$$

$\therefore$ Marginal Supply, when P = 4.

$$\left(\frac{dS}{dP}\right)_{p=4} = 4(4) + 1 = 17$$

EXERCISE 9.2

(I) Differentiate the following functions *w.r.t.x*

(1) $\frac{x}{x+1}$ (2) $\frac{x^2+1}{x}$ (3) $\frac{1}{e^x+1}$

(4) $\frac{e^x}{e^x+1}$ (5) $\frac{x}{\log x}$ (6) $\frac{2^x}{\log x}$

(7) $\frac{(2e^x-1)}{(2e^x+1)}$ (8) $\frac{(x+1)(x-1)}{(e^x+1)}$

II) Solve the following examples:

(1) The demand D for a price P is given as $D = \frac{27}{P}$, find the rate of change of demand when price is 3.

(2) If for a commodity; the price-demand relation is given as $D = \frac{P+5}{P-1}$. Find the marginal demand when price is 2.

(3) The demand function of a commodity is given as $P = 20 + D - D^2$. Find the rate at which price is changing when demand is 3.

(4) If the total cost function is given by; $C = 5x^3 + 2x^2 + 7$; find the average cost and the marginal cost when $x = 4$.

(5) The total cost function of producing n notebooks is given by $C = 1500 - 75n + 2n^2 + \frac{n^3}{5}$. Find the marginal cost at $n = 10$.

(6) The total cost of 't' toy cars is given by $C=5(2^t)+17$. Find the marginal cost and average cost at t=3.

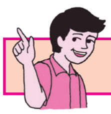
(7) If for a commodity; the demand function is given by, $D = \sqrt{75-3P}$. find the marginal demand function when $P = 5$

(8) The total cost of producing x units is given by $C=10e^{2x}$, find its marginal cost and average cost when $x = 2$

(9) The demand function is given as $P = 175 + 9D + 25D^2$. Find the revenue, average revenue and marginal revenue when demand is 10.

(10) The supply S for a commodity at price P is given by $S = P^2 + 9P - 2$. Find the marginal supply when price is 7.

(11) The cost of producing x articles is given by $C = x^2 + 15x + 81$. Find the average cost and marginal cost functions. Find marginal cost when $x = 10$. Find x for which the marginal cost equals the average cost.



Let's remember!

- If u and v are differentiable functions of x and if

(i) $y = u \pm v$ then $\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$

(ii) $y = u.v$ then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

(iii) $y = \frac{u}{v}$, $v \neq 0$ then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

- Standard Derivatives

$f(x)$	$f'(x)$
x^n	nx^{n-1} for all $n \in N$
x	1
$1/x$	$-1/x^2$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
k (constant)	0
e^x	e^x
a^x	$a^x \log a$
$\log x$	$1/x$

MISCELLANEOUS EXERCISE - 9

I. Differentiate the following functions.*w.r.t.x.*

(1) x^5 (2) x^{-2} (3) $\sqrt{x}$

(4) $x\sqrt{x}$ (5) $\frac{1}{\sqrt{x}}$ (6) 7^x

II. Find $\frac{dy}{dx}$ if

(1) $y = x^2 + \frac{1}{x^2}$ (2) $y = \left(\sqrt{x}+1\right)^2$

(3) $y = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$

(4) $y = x^3 - 2x^2 + \sqrt{x} + 1$

(5) $y = x^2 + 2^x - 1$

(6) $y = (1-x)(2-x)$

(7) $y = \frac{1+x}{2+x}$ (8) $y = \frac{(\log x + 1)}{x}$

(9) $y = \frac{e^x}{\log x}$

(10) $y = x \log x (x^2 + 1)$

III. Solve the following.

(1) The relation between price (P) and demand (D) of a cup of Tea is given by $D = \frac{12}{P}$. Find the rate at which the demand changes when the price is Rs. 2/- Interpret the result.

(2) The demand (D) of biscuits at price P is given by $D = \frac{64}{P^3}$, find the marginal demand when price is Rs. 4/-.

(3) The supply S of electric bulbs at price P is given by $S = 2P^3 + 5$. Find the marginal supply when the price is Rs. 5/- Interpret the result.

(4) The marginal cost of producing x items is given by $C = x^2 + 4x + 4$. Find the average cost and the marginal cost. What is the marginal cost when $x = 7$.

(5) The Demand D for a price P is given as $D = \frac{27}{P}$, Find the rate of change of demand when the price is Rs. 3/-.

(6) If for a commodity; the price demand relation is given be $D = \left(\frac{P+5}{P-1}\right)$. Find the marginal demand when price is Rs. 2/-.

(7) The price function P of a commodity is given as $P = 20 + D - D^2$ where D is demand. Find the rate at which price (P) is changing when demand D = 3.

(8) If the total cost function is given by $C = 5x^3 + 2x^2 + 1$; Find the average cost and the marginal cost when $x = 4$.

(9) The supply S for a commodity at price P is given by $S = P^2 + 9P - 2$. Find the marginal supply when price Rs. 7/-.

(10) The cost of producing x articles is given by $C = x^2 + 15x + 81$. Find the average cost and marginal cost functions. Find the marginal cost when $x = 10$. Find x for which the marginal cost equals the average cost.

Gottfried Wilhelm Leibnitz (1646 – 1716)

Gottfried Wilhelm Leibniz was a prominent German polymath and philosopher in the history of mathematics and the history of philosophy. His most notable accomplishment was conceiving the ideas of differential and integral calculus, independently of Isaac Newton's contemporaneous developments. Calculus was discovered and developed independently by Sir Isaac Newton (1642 - 1727) in England and Gottfried Wilhelm Leibnitz (1646 - 1716) in Germany, towards the end of 17^{th} century.

ACTIVITIES

Activity 9.1:

Find the derivative of $\frac{1}{x^2}$ by first principle.

$$f(x) = \frac{1}{x^2}, \qquad f(x+h) = \frac{1}{(x+h)^2}$$

$$f'(x) = \lim_{h\to 0}\left(\frac{\square - \square}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}\right)$$

$$= \lim_{h\to 0}\frac{\square - \square}{h(x+h)^2 x^2}$$

$$= \lim_{h\to 0}\frac{x^2 - (x^2 + 2xh + h^2)}{h(x+h)^2 . \square}$$

$$= \lim_{h \to 0} \frac{\boxed{}}{h(x+h)^2 . x^2}$$

$$= \frac{-\boxed{}}{\boxed{}} = \_\_\_\_\_\_$$

Activity 9.2:

The supply S of pens at price P is given by $S = 2P^3 + 7$. Find the marginal supply when price is Rs.5/-. Interpret the result.

Given : $S = 2p^3 + 7$

$$\therefore \quad \frac{ds}{dp} = \ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots$$

$\therefore$ Marginal supply when P = 5/-

$$= \left(\frac{ds}{dp}\right)_{p=5} = \boxed{} = \boxed{}$$

Interpretation : ____________________

Activity 9.3:

The relation between price (P) and demand (D) at a cup of Tea is given as $D = \frac{32}{p}$. Find the rate of which the demand changes when the price is Rs. 4/-. Interpret the result.

Given : $D = \frac{32}{p}$

$$\therefore \quad \frac{dD}{dP} = \ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots$$

$$= \left(\frac{dD}{dp}\right)_{P=4} = \boxed{}$$

Interpretation : ____________________

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