

5. LOCUS AND STRAIGHT LINE



Let's study.

- Locus
- Slope of a line
- Perpendicular and parallel lines
- Angle between intersecting lines
- Equations of lines in different forms :
 - Slope point form
 - Slope intercept form
 - Two points form
 - Double intercept form
 - Normal form
 - General form
- Distance of a point from a line
- Distance between parallel lines



Let's recall.

We are familiar with the perpendicular bisector of a segment, the bisector of an angle, circle and triangle etc.

These geometrical figures are sets of points in plane which satisfy certain conditions.

- The perpendicular bisector of a segment in a plane is the set of points in the plane which are equidistant from the end points of the segment. This set is a line.
- The bisector of an angle is the set of points in the plane of the angle which are equidistant from the arms of the angle. This set is a ray.



Let's learn.

5.1 : DEFINITION : LOCUS : A set of points in a plane which satisfy certain geometrical condition (or conditions) is called a locus.

$L = \{P \mid P \text{ is a point in a plane and } P \text{ satisfy given geometrical condition}\}$

Here P is the representative of all points in L . L is called the *locus* of point P . Locus is a set of points. Every locus has corresponding geometrical figure. We may say P is a point in the locus or a point on the locus.

Examples:

- The perpendicular bisector of segment AB is the set $M = \{P \mid P \text{ is a point in a plane such that } PA = PB\}$.
- The bisector of angle AOB is the set :
 $D = \{P \mid P \text{ is a point in the plane such that } P \text{ is equidistant from } OA \text{ and } OB\}$
 $= \{P \mid \angle POA = \angle POB\}$
- The circle with center O and radius 4 is the set $L = \{P \mid OP = 4, P \text{ is a point in the plane}\}$

The plural of locus is loci.

5.2 : EQUATION OF LOCUS : Every point in XY plane has a pair of co-ordinates. If an equation is satisfied by co-ordinates of all points on the locus and if any point whose co-ordinates satisfies the equation is on the locus then that equation is called the equation of the locus.

Ex.1 We know that the y co-ordinate of every point on the X -axis is zero and this is true for points on the X -axis only. Therefore the equation of the X -axis is $y = 0$. Similarly every point on Y -axis will have x -co-ordinate zero and conversely, so its equation will be $x = 0$.

Note: The x -co-ordinate is also called as abscissa and y -co-ordinate is called as ordinate.

Ex.2 Let $L = \{P \mid OP = 4\}$. Find the equation of L .

Solution : L is the locus of points in the plane which are at 4 unit distance from the origin.

Let $P(x, y)$ be any point on the locus L .

$$\text{As } OP = 4, OP^2 = 16$$

$$\therefore (x - 0)^2 + (y - 0)^2 = 16$$

$$\therefore x^2 + y^2 = 16$$

This is the equation of the locus L .

The locus is seen to be a circle

Ex.3 Find the equation of the locus of points which are equidistant from $A(-3, 0)$ and $B(3, 0)$. Identify the locus.

Solution : Let $P(x, y)$ be any point on the required locus.

P is equidistant from A and B .

$$\therefore PA = PB$$

$$\therefore PA^2 = PB^2$$

$$\therefore (x + 3)^2 + (y - 0)^2 = (x - 3)^2 + (y - 0)^2$$

$$\therefore x^2 + 6x + 9 + y^2 = x^2 - 6x + 9 + y^2$$

$$\therefore 12x = 0$$

$$\therefore x = 0. \text{ The locus is the } Y\text{-axis.}$$



Let's learn.

Shift of Origin : Let OX, OY be the co-ordinate axes. Let $O'(h, k)$ be a point in the plane. Let the origin be shifted to O' . Let $O'X', O'Y'$ be the new co-ordinate axes through O' and parallel to the axes OX and OY respectively.

Let (x, y) be the co-ordinates of P referred to the co-ordinates axes OX, OY and (X, Y) be the co-ordinates of P referred to the co-ordinate axes $O'X', O'Y'$. To find relations between (x, y) and (X, Y) .

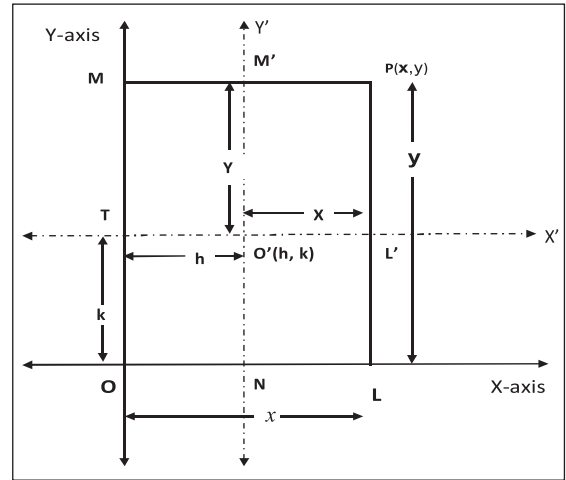


Figure 5.3

Draw $PL \perp OX$ and suppose it intersects $O'X'$ in L' .

Draw $PM \perp OY$ and suppose it intersects $O'Y'$ in M' .

Let $O'Y'$ meet line OX in N and $O'X'$ meet OY in T .

$$\therefore ON = h, OT = k, OL = x, OM = y,$$

$$O'L' = X, O'M' = Y$$

$$\text{Now } x = OL = ON + NL = ON + O'L' = h + X$$

$$\text{and } y = OM = OT + TM = OT + O'M' = k + Y$$

$$\therefore \boxed{x = X + h}, \boxed{y = Y + k}$$

These equations are known as the formulae for shift of origin.

Ex.4 If the origin is shifted to the point $O'(3, 2)$ the directions of the axes remaining the same, find the new co-ordinates of the points

(a) $A(4, 6)$ (b) $B(2, -5)$.

Solution : We have $(h, k) = (3, 2)$

$$x = X + h, y = Y + k$$

$$\therefore x = X + 3, \text{ and } y = Y + 2 \dots\dots\dots (1)$$

$$(a) (x, y) = (4, 6)$$

$$\therefore \text{From (1), we get } 4 = X + 3, 6 = Y + 2$$

$$\therefore X = 1 \text{ and } Y = 4.$$

New co-ordinates of A are $(1, 4)$

$$(ii) (x, y) = (2, -5)$$

$$\text{from (1), we get } 2 = X + 3, -5 = Y + 2$$

$$\therefore X = -1 \text{ and } Y = -7.$$

New co-ordinates of B are $(-1, -7)$

Ex.5 The origin is shifted to the point $(-2, 1)$, the axes being parallel to the original axes. If the new co-ordinates of point A are $(7, -4)$, find the old co-ordinates of point A.

Solution : We have $(h, k) = (-2, 1)$

$$x = X + h, y = Y + k$$

$$\therefore x = X - 2, y = Y + 1$$

$$(X, Y) = (7, -4)$$

$$\text{we get } x = 7 - 2 = 5,$$

$$y = -4 + 1 = -3.$$

$$\therefore \text{Old co-ordinates A are } (5, -3)$$

Ex.6 Obtain the new equation of the locus $x^2 - xy - 2y^2 - x + 4y + 2 = 0$ when the origin is shifted to $(2, 3)$, the directions of the axes remaining the same.

Solution : Here $(h, k) = (2, 3)$

$$\therefore x = X + h, y = Y + k \text{ gives}$$

$$\therefore x = X + 2, y = Y + 3$$

The given equation

$$x^2 - xy - 2y^2 - x + 4y + 2 = 0 \text{ becomes}$$

$$(X+2)^2 - (X+2)(Y+3) - 2(Y+3)^2$$

$$- (X+2) + 4(Y+3) + 2 = 0$$

$$\therefore X^2 - XY - 2Y^2 - 10Y - 8 = 0$$

This is the new equation of the given locus.

EXERCISE 5.1

1. If $A(1, 3)$ and $B(2, 1)$ are points, find the equation of the locus of point P such that $PA = PB$.

2. $A(-5, 2)$ and $B(4, 1)$. Find the equation of the locus of point P, which is equidistant from A and B.
3. If $A(2, 0)$ and $B(0, 3)$ are two points, find the equation of the locus of point P such that $AP = 2BP$.
4. If $A(4, 1)$ and $B(5, 4)$, find the equation of the locus of point P if $PA^2 = 3PB^2$.
5. $A(2, 4)$ and $B(5, 8)$, find the equation of the locus of point P such that $PA^2 - PB^2 = 13$.
6. $A(1, 6)$ and $B(3, 5)$, find the equation of the locus of point P such that segment AB subtends right angle at P. ($\angle APB = 90^\circ$)
7. If the origin is shifted to the point $O'(2, 3)$, the axes remaining parallel to the original axes, find the new co-ordinates of the points
(a) $A(1, 3)$ (b) $B(2, 5)$
8. If the origin is shifted to the point $O'(1, 3)$ the axes remaining parallel to the original axes, find the old co-ordinates of the points
(a) $C(5, 4)$ (b) $D(3, 3)$
9. If the co-ordinates $(5, 14)$ change to $(8, 3)$ by shift of origin, find the co-ordinates of the point where the origin is shifted.
10. Obtain the new equations of the following loci if the origin is shifted to the point $O'(2, 2)$, the direction of axes remaining the same :
(a) $3x - y + 2 = 0$
(b) $x^2 + y^2 - 3x = 7$
(c) $xy - 2x - 2y + 4 = 0$

5.3 LINE :

The aim of this chapter is to study a line and its equation. The locus of a point in a plane such that the segment joining any two points on the locus lies completely on the locus is called a line.

The simplest locus in a plane is a line. The characteristic property of this locus is that if we find the slope of a segment joining any two points on this locus, then the slope is constant.

Further we know that if a line meets the X-axis in the point A ($a, 0$), then a is called the x-intercept of the line. If it meets the Y-axis in the point B ($0, b$) then b is called the y-intercept of the line.



Let's learn.

5.3.1 : Inclination of a line : The smallest angle made by a line with the positive direction of the X-axis measured in anticlockwise sense is called the inclination of the line. We denote inclination by θ . Clearly $0^\circ < \theta < 180^\circ$.

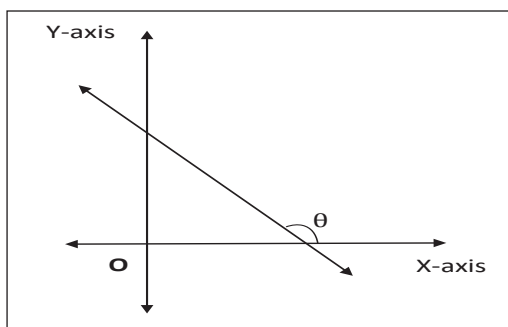


Figure 5.1

Remark : Two lines are parallel if and only if they have the same inclination.

The inclination of the X-axis and a line parallel to the X-axis is Zero. The inclination of the Y-axis and a line parallel to the Y-axis is 90° .

5.3.2 : Slope of a line : If θ is the inclination of a line then $\tan\theta$ (if it exists) is called the slope of the line.

Let $A(x_1, y_1)$, $B(x_2, y_2)$ be any two points on a non-vertical line whose inclination is θ then verify that

$$\tan \theta = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_1 \neq x_2.$$

The slope of the Y-axis is not defined. Similarly the slope of a line parallel to the Y-axis is not defined. The slope of the X-axis is 0. The

slope of a line parallel to the X-axis is also 0.

Remark : Two lines are parallel if and only if they have the same slope.

Ex.1 Find the slope of the line whose inclination is 60° .

Solution : The tangent ratio of the inclination of a line is called the slope of the line.

$$\text{Inclination } \theta = 60^\circ.$$

$$\therefore \text{ slope} = \tan\theta = \tan 60^\circ = \sqrt{3}.$$

Ex.2 Find the slope of the line which passes through the points A(2, 4) and B(5, 7).

Solution : The slope of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$\frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Slope of the line AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 4}{5 - 2} = 1$$

Note that $x_1 \neq x_2$.

Ex.3 Find the slope of the line which passes through the origin and the point A(-4, 4).

Solution : The slope of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}. \text{ Here } A(-4, 4) \text{ and } O(0, 0).$$

$$\text{Slope of the line OA} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 4}{0 + 4} = -1.$$

Note that $x_1 \neq x_2$.



Let's study.

5.3.3 : Perpendicular Lines : Lines having slopes m_1 and m_2 are perpendicular to each other if and only if $m_1 \times m_2 = -1$.

Ex.1 Show that line AB is perpendicular to line BC, where A(1, 2), B(2, 4) and C(0, 5).

Solution : Let slopes of lines AB and BC be m_1 and m_2 respectively.

$$\therefore m_1 = \frac{4 - 2}{2 - 1} = 2 \text{ and}$$

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 4}{0 - 2} = -\frac{1}{2}$$

$$\text{Now } m_1 \times m_2 = 2 \times \left(-\frac{1}{2}\right) = -1$$

$\therefore$ Line AB is perpendicular to line BC.

Ex.2 A(1,2), B(2,3) and C(-2,5) are the vertices of a ΔABC . Find the slope of the altitude drawn from A.

Solution : The slope of line BC is

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3}{-2 - 2} = -\frac{2}{4} = -\frac{1}{2}$$

Altitude drawn from A is perpendicular to BC.

If m_2 is the slope of the altitude from A then $m_1 \times m_2 = -1$.

$$\therefore m_2 = \frac{-1}{m_1} = 2.$$

The slope of the altitude drawn from A is 2.

5.3.4 : Angle between intersecting lines :

If θ is the acute angle between non-vertical lines having slopes m_1 and m_2 then

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|, \text{ where } 1 + m_1 m_2 \neq 0$$

Ex.1 Find the acute angle between lines having slopes 3 and -2.

Solution : Let $m_1 = 3$ and $m_2 = -2$.

Let θ be the acute angle between them.

$$\therefore \tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = \left| \frac{3 - (-2)}{1 + (3)(-2)} \right| = \left| \frac{5}{-5} \right| = 1$$

$$\therefore \theta = 45^\circ$$

The acute angle between lines having slopes 3 and -2 is 45° .

Ex.2 If the angle between two lines is 45° and the slope of one of the lines is $\frac{1}{2}$, find the slope of the other line.

Solution : If θ is the acute angle between lines having slopes m_1 and m_2 then

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

Given $\theta = 45^\circ$.

Let $m_1 = \frac{1}{2}$. Let m_2 be slope of the other line.

$$\tan 45^\circ = \left| \frac{\frac{1}{2} - m_2}{1 + \left(\frac{1}{2}\right)m_2} \right| \quad \therefore 1 = \left| \frac{1 - 2m_2}{2 + m_2} \right|$$

$$\therefore \frac{1 - 2m_2}{2 + m_2} = 1 \quad \text{or} \quad \frac{1 - 2m_2}{2 + m_2} = -1$$

$$\therefore 1 - 2m_2 = 2 + m_2 \quad \text{or} \quad 1 - 2m_2 = -2 - m_2$$

$$\therefore 3m_2 = -1 \quad \text{or} \quad m_2 = 3$$

$$\therefore m_2 = 3 \quad \text{or} \quad -\frac{1}{3}$$

EXERCISE 5.2

- Find the slope of each of the following lines which pass through the points :
(a) (2, -1), (4, 3) (b) (-2, 3), (5, 7)
(c) (2, 3), (2, -1) (d) (7, 1), (-3, 1)
- If the X and Y-intercepts of line L are 2 and 3 respectively then find the slope of line L.
- Find the slope of the line whose inclination is 30° .
- Find the slope of the line whose inclination is 45° .
- A line makes intercepts 3 and 3 on co-ordinate axes. Find the inclination of the line.
- Without using Pythagoras theorem show that points A(4,4), B(3, 5) and C(-1, -1) are the vertices of a right angled triangle.
- Find the slope of the line which makes angle of 45° with the positive direction of the Y-axis measured clockwise.

8. Find the value of k for which points $P(k, -1)$, $Q(2, 1)$ and $R(4, 5)$ are collinear.



Let's learn.

5.4: EQUATIONS OF LINES IN DIFFERENT FORMS :

We know that line is a locus. Every locus has an equation. An equation in x and y which is satisfied by the co-ordinates of all points on a line and which is not satisfied by the co-ordinates of any point which does not lie on the line is called the equation of the line.

The equation of any line parallel to the Y-axis is of the type $x = k$ (where k is a constant) and the equation of any line parallel to the X-axis is of the type $y = k$. This is all about vertical and horizontal lines.

Let us obtain equations of non-vertical and non-horizontal lines in different forms:

5.4.1 : Slope-Point Form : To find the equation of a line having slope m and which passes through the point $A(x_1, y_1)$.

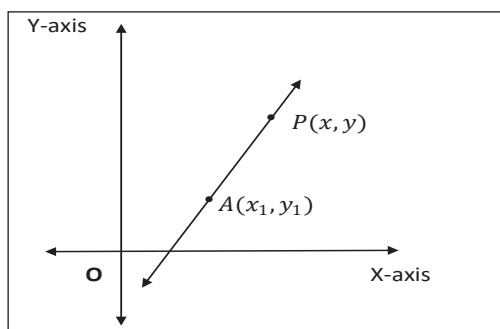


Figure 5.9

The equation of the line having slope m and passing through $A(x_1, y_1)$ is $(y - y_1) = m(x - x_1)$.

Remark : In particular if the line passes through the origin $O(0, 0)$ and has slope m , then its equation is $y - 0 = m(x - 0)$

$$\text{i.e. } y = mx$$

Ex.1) Find the equation of the line passing through the point $A(2, 1)$ and having slope -3 .

Solution : Given line passes through the point $A(2, 1)$ and slope of the line is -3 .

The equation of the line having slope m and passing through $A(x_1, y_1)$ is $(y - y_1) = m(x - x_1)$.

$\therefore$ the equation of the required line is

$$y - 1 = -3(x - 2)$$

$$\therefore y - 1 = -3x + 6$$

$$\therefore 3x + y - 7 = 0$$

5.4.2 : Slope-Intercept form : The equation of a line having slope m and which makes intercept c on the Y-axis is $y = mx + c$.

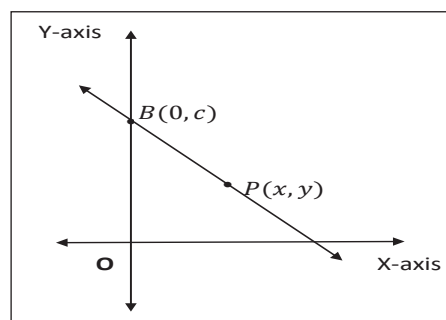


Figure 5.10

Ex.2 Obtain the equation of the line having slope 3 and which makes intercept 4 on the Y-axis.

Solution: The equation of line having slope m and which makes intercept c on the Y-axis is

$$y = mx + c.$$

$\therefore$ the equation of the line giving slope 3 and making y-intercept 4 is $y = 3x + 4$.

5.4.3 : Two-points Form : The equation of a line which passes through points $A(x_1, y_1)$ and

$$B(x_2, y_2) \text{ is } \frac{y - y_1}{x - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

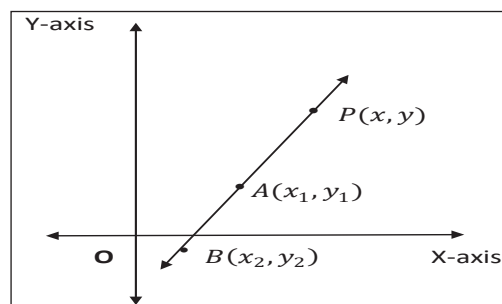


Figure 5.11

Ex.3 Obtain the equation of the line passing through points A(2, 1) and B(1, 2).

Solution : The equation of the line which passes through points A(x_1, y_1) and B(x_2, y_2) is

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}.$$

$\therefore$ The equation of the line passing through points A(2, 1) and B(1, 2) is $\frac{x-2}{1-2} = \frac{y-1}{2-1}$

$$\therefore \frac{x-2}{-1} = \frac{y-1}{1}$$

$$\therefore x - 2 = -y + 1$$

$$\therefore x + y - 3 = 0$$

5.4.4 : Double-Intercept form : The equation of the line which makes non-zero intercepts a and b on the X and Y axes respectively is

$$\frac{x}{a} + \frac{y}{b} = 1$$

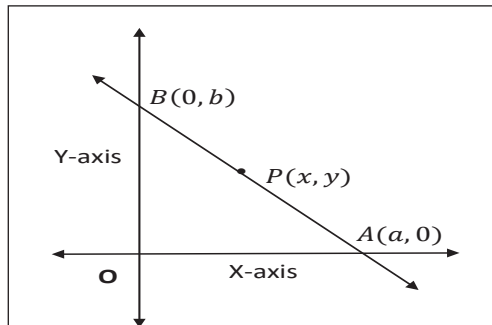


Figure 5.12

Ex.1 Obtain the equation of the line which makes intercepts 3 and 4 on the X and Y axes respectively.

Solution : The equation of the line which makes intercepts a and b on the X and Y

co-ordinate axes $\frac{x}{a} + \frac{y}{b} = 1$

The equation of the line which makes intercepts 3 and 4 on the co-ordinate axes is $\frac{x}{3} + \frac{y}{4} = 1$
 $\therefore 4x + 3y - 12 = 0.$

5.4.5 : Normal Form : Let L be a line and segment ON be the perpendicular (normal) drawn from the origin to line L.

If ON = p . Let the ray ON make angle α with the positive X-axis.

$$\therefore N = (p \cos \alpha, p \sin \alpha)$$

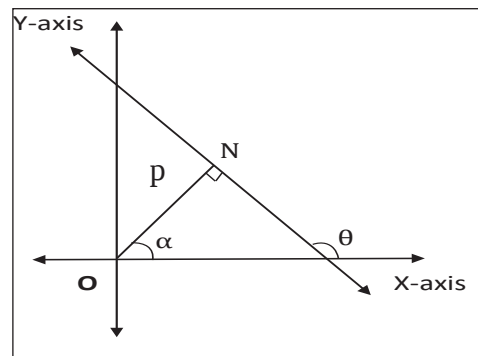


Figure 5.13

Let θ be the inclination of the line L. The equation of the line, the normal to which from the origin has length p and the normal makes angle α with the positive directions of the X-axis, is $x \cos \alpha + y \sin \alpha = p$.

Ex.1 The perpendicular drawn from the origin to a line has length 5 and the perpendicular makes angle 30° with the positive direction of the X-axis. Find the equation of the line.

Solution : The perpendicular (normal) drawn from the origin to the line has length 5.

$$\therefore p = 5$$

The perpendicular (normal) makes angle 30° with the positive direction of the X-axis.

$$\therefore \theta = 30^\circ$$

The equation of the required line is

$$x \cos \alpha + y \sin \alpha = p$$

$$\therefore x \cos 30^\circ + y \sin 30^\circ = 5$$

$$\therefore \frac{\sqrt{3}}{2}x + \frac{1}{2}y = 5$$

$$\therefore \sqrt{3}x + y - 10 = 0$$

Ex.2 Reduce the equation $\sqrt{3}x - y - 2 = 0$ into normal form. Find the values of p and α .

Solution : Comparing $\sqrt{3}x - y - 2 = 0$ with $ax + by + c = 0$ we get $a = \sqrt{3}$, $b = -1$ and $c = -2$.

$$\sqrt{a^2 + b^2} = \sqrt{3+1} = 2$$

Divide the given equation by 2.

$$\frac{\sqrt{3}}{2}x - \frac{1}{2}y = 1$$

$\therefore \cos 30^\circ x - \sin 30^\circ y = 1$ is the required normal form of the given equation.
 $p = 1$ and $\theta = 30^\circ$

SOLVED EXAMPLES

Find the equation of the line :

- parallel to the X-axis and 3 unit below it,
- passing through the origin and having inclination 30°
- passing through the point $A(5,2)$ and having slope 6.
- passing through the points $A(2,-1)$ and $B(5,1)$
- having slope $-\frac{3}{4}$ and y -intercept 5,
- making intercepts 3 and 6 on the X and Y axes respectively.

Solution :

- Equation of a line parallel to the X-axis is of the form $y = k$,

$\therefore$ the equation of the required line is $y = -3$

- Equation of a line through the origin and having slope m is of the form $y = mx$.

$$\text{Here, } m = \tan 30^\circ = \frac{1}{\sqrt{3}}.$$

$\therefore$ the equation of the required line is

$$y = \frac{1}{\sqrt{3}} x$$

- By using the point-slope form is

$$y - y_1 = m(x - x_1)$$

equation of the required line is

$$(y-2) = 6(x-5)$$

$$\text{i.e. } 6x - y - 28 = 0$$

- Here $(x_1, y_1) = (2,-1)$; $(x_2, y_2) = (5,1)$

By using the two points form

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

the equation of the required line is

$$\frac{x-2}{5-2} = \frac{y+1}{1+1}$$

$$\therefore 2(x-2) = 3(y+1)$$

$$\therefore 2x - 3y - 7 = 0$$

- Given $m = -\frac{3}{4}$, $c = 5$

By using the slope intercept form $y = mx + c$

the equation of the required line is

$$y = -\frac{3}{4}x + 5 \quad \therefore 3x + 4y - 20 = 0$$

- (vi) x -intercept $= a = 3$;
 y -intercept $= b = 6$.

By using the double intercept form

$$\frac{x}{a} + \frac{y}{b} = 1$$

the equation of the required line is

$$\frac{x}{3} + \frac{y}{6} = 1$$

$$2x + y - 6 = 0$$

EXERCISE 5.3

- Write the equation of the line :
 - parallel to the X -axis and at a distance of 5 unit from it and above it.
 - parallel to the Y -axis and at a distance of 5 unit from it and to the left of it.
 - parallel to the X -axis and at a distance of 4 unit from the point $(-2, 3)$.
- Obtain the equation of the line :
 - parallel to the X -axis and making an intercept of 3 unit on the Y -axis.
 - parallel to the Y -axis and making an intercept of 4 unit on the X -axis.
- Obtain the equation of the line containing the point :
 - $A(2, -3)$ and parallel to the Y -axis.
 - $B(4, -3)$ and parallel to the X -axis.
- Find the equation of the line passing through the points $A(2, 0)$ and $B(3, 4)$.
- Line $y = mx + c$ passes through points $A(2, 1)$ and $B(3, 2)$. Determine m and C .
- The vertices of a triangle are $A(3, 4)$, $B(2, 0)$ and $C(1, 6)$ Find the equations of
 - side BC
 - the median AD
 - the line passing through the mid points of sides AB and BC .
- Find the X and Y intercepts of the following lines :
 - $\frac{x}{3} + \frac{y}{2} = 1$
 - $\frac{3x}{2} + \frac{2y}{3} = 1$
 - $2x - 3y + 12 = 0$
- Find the equations of the lines containing the point $A(3, 4)$ and making equal intercepts on the co-ordinates axes.
- Find the equations of the altitudes of the triangle whose vertices are $A(2, 5)$, $B(6, -1)$ and $C(-4, -3)$.

5.5 : General form of equation of line: We can write equation of every line in the form $ax + by + c = 0$. where $a, b, c \in \mathbb{R}$ and all are not simultaneously zero.

This form of equation of a line is called the general form.

The general form of $y = 3x + 2$ is $3x - y + 2 = 0$

The general form of $\frac{x}{2} + \frac{y}{3} = 1$ is $3x + 2y - 6 = 0$

The slope of the line $ax + by + c = 0$ is $-\frac{a}{b}$.

the x -intercept is $-\frac{c}{a}$ and

the y -intercept is $-\frac{c}{b}$.

Ex.1 Find the slope and intercepts made by the following lines :

- $x + y + 10 = 0$
- $2x + y + 30 = 0$
- $x + 3y - 15 = 0$

Solution:

(a) Comparing equation $x+y+10=0$

with $ax + by + c = 0$,

we get $a = 1, b = 1, c = 10$

$$\therefore \text{Slope of this line} = -\frac{a}{b} = -1$$

$$\text{The X-intercept is } -\frac{c}{a} = -\frac{10}{1} = -10$$

$$\text{The Y-intercept is } = -\frac{c}{b} = -\frac{10}{1} = -10$$

(b) Comparing the equation $2x+y+30=0$

with $ax + by + c = 0$.

we get $a = 2, b = 1, c = 30$

$$\therefore \text{Slope of this line} = -\frac{a}{b} = -2$$

$$\text{The X-intercept is } -\frac{c}{a} = -\frac{30}{2} = -15$$

$$\text{The Y-intercept is } -\frac{c}{b} = -\frac{30}{1} = -30$$

(c) Comparing equation $x + 3y - 15 = 0$ with

$ax + by + c = 0$.

we get $a = 1, b = 3, c = -15$

$$\therefore \text{Slope of this line} = -\frac{a}{b} = -\frac{1}{3}$$

$$\text{The x-intercept is } -\frac{c}{a} = -\frac{-15}{1} = 15$$

$$\text{The y-intercept is } -\frac{c}{b} = -\frac{-15}{3} = 5$$

Ex.2 Find the acute angle between the following pairs of lines :

a) $12x-4y=5$ and $4x+2y=7$

b) $y=2x+3$ and $y=3x+7$

Solution :

(a) Slopes of lines $12x-4y=5$ and

$$4x + 2y = 7 \text{ are } m_1 = 3 \text{ and } m_2 = -2.$$

If θ is the acute angle between lines having slope m_1 and m_2 then

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\therefore \tan \theta = \left| \frac{3 - (-2)}{1 + (3)(-2)} \right| = \left| \frac{5}{-5} \right| = 1$$

$$\therefore \tan \theta = 1 \quad \therefore \theta = 45^\circ$$

(b) Slopes of lines $y=2x+3$ and $y=3x+7$

$$\text{are } m_1 = 2 \text{ and } m_2 = 3$$

The acute angle θ between lines having slopes m_1 and m_2 is given by

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\therefore \tan \theta = \left| \frac{2-3}{1+(2)(3)} \right| = \left| \frac{-1}{7} \right| = \frac{1}{7}$$

$$\therefore \tan \theta = \frac{1}{7}.$$

Ex.3 Find the acute angle between the lines

$$y - \sqrt{3}x + 1 = 0 \text{ and } \sqrt{3}y - x + 7 = 0.$$

Solution : Slopes of the given lines are

$$m_1 = \sqrt{3} \text{ and } m_2 = \frac{1}{\sqrt{3}}.$$

The acute angle θ between lines having slopes m_1 and m_2 is given by

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\therefore \tan \theta = \left| \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}} \right| = \left| \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1+1} \right| = \left| \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{2} \right|$$

$$= \left| \frac{3-1}{2\sqrt{3}} \right| = \frac{1}{\sqrt{3}}$$

$$= \left| \frac{2}{2\sqrt{3}} \right| = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = 30^\circ$$

Ex.4 Show that following pairs of lines are perpendicular to each other.

a) $2x-4y=5$ and $2x+y=17$.

b) $y=2x+23$ and $2x+4y=27$

Solution :

(i) Slopes of lines $2x-4y=5$ and

$$2x+y=17 \text{ are } m_1 = \frac{1}{2} \text{ and } m_2 = -2$$

$$\text{Since } m_1 \cdot m_2 = \frac{1}{2} \times (-2) = -1,$$

given lines are perpendicular to each other.

(ii) Slopes of lines $y = 2x + 23$ and

$$2x+4y = 27 \text{ are } m_1 = -\frac{1}{2} \text{ and } m_2 = 2.$$

$$\text{Since } m_1 \cdot m_2 = -\frac{1}{2} \times (2) = -1, \text{ given lines are perpendicular to each other.}$$

Ex.5 Find equations of lines which pass through the origin and make an angle of 45° with the line $3x - y = 6$.

Solution : Slope of the line $3x - y = 6$ is 3. Let m be the slope of one of the required line. The angle between these lines is 45° .

$$\therefore \tan 45^\circ = \left| \frac{m-3}{1+(m)(3)} \right|$$

$$\therefore 1 = \left| \frac{m-3}{1+3m} \right| \quad \therefore |1+3m| = |m-3|$$

$$\therefore 1+3m = m-3 \quad \text{or} \quad 1+3m = -(m-3)$$

$$\therefore m = -2 \text{ or } \frac{1}{2}$$

Slopes of required lines are $m_1 = -2$ and

$$m_2 = \frac{1}{2}$$

Required lines pass through the origin.

$\therefore$ Their equations are $y = -2x$ and

$$y = \frac{1}{2}x$$

$$\therefore 2x+y=0 \text{ and } x-2y=0$$

Ex.6 A line is parallel to the line $2x+y=7$ and passes through the origin. Find its equation.

Solution : Slope of the line $2x+y=7$ is -2 .

$\therefore$ slope of the required line is also -2

Required line passes through the origin.

$\therefore$ Its equation is $y = -2x$

$$\therefore 2x+y=0.$$

Ex.7 A line is parallel to the line $x+3y=9$ and passes through the point $A(2,7)$. Find its equation.

Solution : Slope of the line $x+3y=9$ is $-\frac{1}{3}$

$\therefore$ slope of the required line is $-\frac{1}{3}$

Required line passes through the point $A(2,7)$.

$\therefore$ Its equation is given by the formula

$$(y - y_1) = m (x - x_1)$$

$$\therefore (y-7) = -\frac{1}{3}(x-2)$$

$$\therefore 3y - 21 = -x + 2$$

$$\therefore x+3y=23.$$

Ex.8 A line is perpendicular to the line $3x+2y-1=0$ and passes through the point $A(1,1)$. Find its equation.

Solution : Slope of the line $3x + 2y - 1 = 0$ is $-\frac{3}{2}$

Required line is perpendicular to it.

The slope of the required line is $\frac{2}{3}$.

Required line passes through the point $A(1,1)$.

$\therefore$ Its equation is given by the formula

$$(y - y_1) = m(x - x_1)$$

$$\therefore (y - 1) = \frac{2}{3}(x - 1)$$

$$\therefore 3y - 3 = 2x - 2$$

$$\therefore 2x - 3y + 1 = 0.$$

5.5.1 : Point of intersection of lines : The co-ordinates of the point of intersection of two intersecting lines can be obtained by solving their equations simultaneously.

SOLVED EXAMPLES

Ex.1 Find the co-ordinates of the point of intersection of lines $x + 2y = 3$ and $2x - y = 1$.

Solution : Solving equations $x + 2y = 3$

and $2x - y = 1$ simultaneously, we get $x = 1$ and $y = 1$.

$\therefore$ Given lines intersect in point $(1, 1)$

Ex.2 Find the equation of line which is parallel to the X-axis and which passes through the point of intersection of lines $x + 2y = 6$ and $2x - y = 2$

Solution : Solving equations $x + 2y = 6$ and $2x - y = 2$ simultaneously, we get $x = 2$ and $y = 2$.

$\therefore$ The required line passes through the

point $(2, 2)$ and has slope 0.

(As it is parallel to the X-axis)

$\therefore$ Its equation is given by $(y - y_1) = m(x - x_1)$
 $(y - 2) = 0(x - 2)$

$\therefore y = 2$.



Let's learn.

5.5.2 : The distance of the Origin from a Line :

The distance of the origin from the line

$$ax + by + c = 0 \text{ is given by } p = \left| \frac{c}{\sqrt{a^2 + b^2}} \right|$$

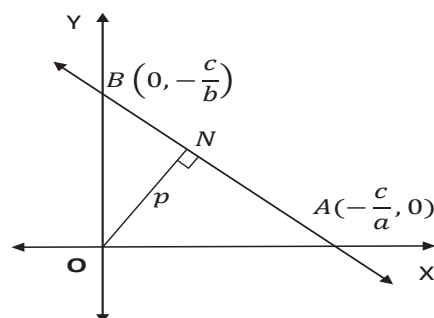


Figure 5.14

5.5.3 : The distance of the point (x_1, y_1) from a line: The distance of the point $P(x_1, y_1)$ from line $ax + by + c = 0$ is given by

$$p = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

5.5.4 : The distance between two parallel lines :

The distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is given

$$\text{by } p = \left| \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right|$$

SOLVED EXAMPLES

Ex.1 Find the distance of the origin from the line $3x+4y+15=0$

Solution : The distance of the origin from the line $ax + by + c = 0$ is given by

$$p = \left| \frac{c}{\sqrt{a^2 + b^2}} \right|$$

$\therefore$ The distance of the origin from the line $3x+4y+15=0$ is given by

$$p = \left| \frac{15}{\sqrt{3^2 + 4^2}} \right| = \frac{15}{5} = 3$$

Ex.2 Find the distance of the point $P(2,5)$ from the line $3x+4y+14=0$

Solution : The distance of the point $P(x_1, y_1)$ from the line $ax + by + c = 0$ is given by

$$p = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

$\therefore$ The distance of the point $P(2,5)$ from the line $3x+4y+14=0$ is given by

$$p = \left| \frac{3(2) + 4(5) + 14}{\sqrt{3^2 + 4^2}} \right| = \frac{40}{5} = 8$$

Ex.3 Find the distance between the parallel lines $6x+8y+21=0$ and $3x+4y+7=0$.

Solution : We write equation $3x+4y+7=0$ as $6x+8y+14=0$ in order to make the coefficients of x and coefficients of y in both equations to be same.

Now by using formula
$$p = \left| \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right|$$

We get the distance between the given parallel lines as

$$p = \left| \frac{21-14}{\sqrt{6^2 + 8^2}} \right| = \frac{7}{10}$$



Let's remember!

- **Locus :** A set of points in a plane which satisfy certain geometrical condition (or conditions) is called a locus.
- **Equation of Locus :** Every point in XY plane has Cartesian co-ordinates. An equation which is satisfied by co-ordinates of all points on the locus and which is not satisfied by the co-ordinates of any point which does not lie on the locus is called the equation of the locus.
- **Inclination of a line :** The smallest angle θ made by a line with the positive direction of the X-axis, measured in anticlockwise sense, is called the inclination of the line. Clearly $0^\circ < \theta < 180^\circ$.
- **Slope of a line :** If θ is the inclination of a line then $\tan\theta$ (if it exists) is called the slope of the line.
If $A(x_1, y_1)$, $B(x_2, y_2)$ are any two points on the line whose inclination is θ then
$$\tan\theta = \frac{y_2 - y_1}{x_2 - x_1} \quad (\text{if } x_1 \neq x_2)$$
- **Perpendicular and parallel lines :** Non-vertical lines having slopes m_1 and m_2 are **perpendicular** to each other if and only if $m_1 m_2 = -1$.

Two lines are **parallel** if and only if they have the same slope, that is $m_1 = m_2$.

- **Angle between intersecting lines :** If θ is the acute angle between lines having slopes

$$m_1 \text{ and } m_2 \text{ then } \tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

provided $m_1 m_2 + 1 \neq 0$.

- **Equations of line in different forms :**

- **Slope point form :** $(y - y_1) = m(x - x_1)$

- **Slope intercept form :** $y = mx + c$

- **Two points form :** $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$

- **Double intercept form :** $\frac{x}{a} + \frac{y}{b} = 1$

- **Normal form :** $x \cos \alpha + y \sin \alpha = p$

- **General form :** $ax + by + c = 0$

- **Distance of a point from a line :**

- **The distance of the origin from the line**

$$ax + by + c = 0 \text{ is given by } p = \left| \frac{c}{\sqrt{a^2 + b^2}} \right|$$

- **The distance of the point $P(x_1, y_1)$ from line $ax + by + c = 0$ is given by**

$$p = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

- **The distance between the Parallel lines:**

The distance between the parallel lines

$$ax + by + c_1 = 0 \text{ and } ax + by + c_2 = 0$$

$$\text{is give by } p = \left| \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right|$$

EXERCISE 5.4

- 1) Find the slope, x-intercept, y-intercept of each of the following lines.

a) $2x + 3y - 6 = 0$

b) $x + 2y = 0$

- 2) Write each of the following equations in $ax + by + c = 0$ form.

a) $y = 2x - 4$ b) $y = 4$

c) $\frac{x}{2} + \frac{y}{4} = 1$ d) $\frac{x}{3} = \frac{y}{2}$

- 3) Show that lines $x - 2y - 7 = 0$ and $2x - 4y + 5 = 0$ are parallel to each other.

- 4) If the line $3x + 4y = p$ makes a triangle of area 24 square unit with the co-ordinate axes then find the value of p .

- 5) Find the co-ordinates of the circumcenter of the triangle whose vertices are $A(-2, 3)$, $B(6, -1)$, $C(4, 3)$.

- 6) Find the equation of the line whose X-intercept is 3 and which is perpendicular to the line $3x - y + 23 = 0$.

- 7) Find the distance of the point $A(-2, 3)$ from the line $12x - 5y - 13 = 0$.

- 8) Find the distance between parallel lines $9x + 6y - 7 = 0$ and $9x + 6y - 32 = 0$.

- 9) Find the equation of the line passing through the point of intersection of lines $x + y - 2 = 0$ and $2x - 3y + 4 = 0$ and making intercept 3 on the X-axis.

- 10) $D(-1, 8)$, $E(4, -2)$, $F(-5, -3)$ are midpoints of sides BC, CA and AB of $\triangle ABC$. Find

(i) equations of sides of $\triangle ABC$.

(ii) co-ordinates of the circumcenter of $\triangle ABC$.

MISCELLANEOUS EXERCISE - 5

1. Find the slopes of the lines passing through the following of points :
(a) (1, 2), (3, -5) (b) (1, 3), (5, 2)
(c) (-1, 3), (3, -1) (d) (2, -5), (3, -1)
2. Find the slope of the line which
 - a) makes an angle of 30° with the positive X-axis.
 - b) makes intercepts 3 and -4 on the axes.
 - c) passes through the points A(-2,1) and the origin .
3. Find the value of k
 - a) if the slope of the line passing through the points (3, 4), (5, k) is 9.
 - b) the points (1, 3), (4, 1), (3, k) are collinear
 - c) the point P(1, k) lies on the line passing through the points A(2, 2) and B(3, 3).
4. Reduce the equation $6x+3y+8=0$ into slope-intercept form. Hence find its slope.
5. Verify that A(2, 7) is not a point on the line $x+2y+2=0$.
6. Find the X-intercept of the line $x+2y-1=0$.
7. Find the slope of the line $y-x+3=0$.
8. Does point A(2,3) lie on the line $3x+2y-6=0$? Give reason.
9. Which of the following lines pass through the origin ?
(a) $x=2$ (b) $y=3$
(c) $y=x+2$ (d) $2x-y=0$
10. Obtain the equation of the line which is :
 - a) parallel to the X-axis and 3 unit below it.
 - b) parallel to the Y-axis and 2 unit to the left of it.
 - c) parallel to the X-axis and making an intercept of 5 on the Y-axis.
 - d) parallel to the Y-axis and making an intercept of 3 on the X-axis.
11. Obtain the equation of the line containing the point
 - a) (2,3) and parallel to the X-axis.
 - b) (2,4) and perpendicular to the Y-axis.
 - c) (2,5) and perpendicular to the X-axis.
12. Find the equation of the line :
 - a) having slope 5 and containing point A(-1,2).
 - b) containing the point (2, 1) and having slope 13.
 - c) containing the point T(7,3) and having inclination 90° .
 - d) containing the origin and having inclination 90° .
 - e) through the origin which bisects the portion of the line $3x+2y=2$ intercepted between the co-ordinate axes.

13. Find the equation of the line passing through the points A(-3,0) and B(0,4).
14. Find the equation of the line :
- having slope 5 and making intercept 5 on the X-axis.
 - having an inclination 60° and making intercept 4 on the Y-axis.
15. The vertices of a triangle are A(1,4), B(2,3) and C(1,6). Find equations of
- the sides
 - the medians
 - Perpendicular bisectors of sides
 - altitudes of $\triangle ABC$.

ACTIVITIES

Activity 5.1 :

Complete the activity and decide whether the line $2x - 4y = 5$ and $2x + y = 17$ are perpendicular to each other.

Slope of line $2x - 4y = 5$ is .

Slope of line $2x + y = 17$ is .

Product of slopes of these line = .

$\therefore$ The given lines are .

Activity 5.2 :

Complete the activity and obtain the equation of the line having slope 2 and which makes intercept 3 on the Y-axis.

Here $c = \text{}$, $m = \text{}$

The equation of line $y = \text{} + c$

$\therefore$ The equation of line is

Activity 5.3 :

Complete the activity and find the equation of the line which is parallel to the line $3x + y = 5$ and passes through the origin.

Slope of line $3x + y = 5$ is

The required line passes through the origin

It's equation is $y = \text{}$

The equation of required line is .

Activity 5.4 :

Complete the following activity and find the value of p, if the area of the triangle formed by the axes and the line $3x + 4y = P$, is 24.

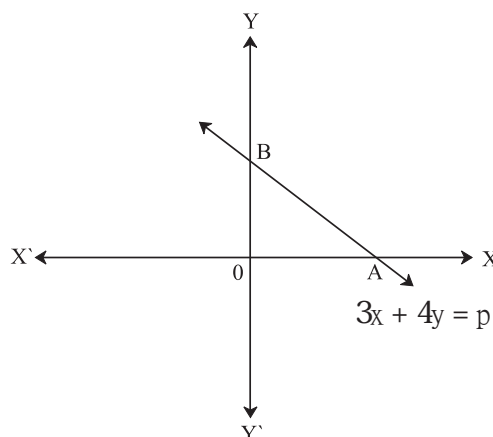


Fig. 5.15

Here, $A = (\text{ $B = (0, \text{$$

$$A(\triangle AOB) = \frac{1}{2} (OA) (OB)$$

$$\therefore 24 = \frac{1}{2} \text{} \text{}$$

$$\therefore P^2 = \text{}$$

$$\therefore P = \text{}$$

